

Nonlinear Schrödinger Equations and N=1 Superconformal Algebra

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Abstract

By using AKNS scheme and soliton connection taking values in N=1 superconformal algebra we obtain new coupled super Nonlinear Schrödinger equations.

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1 Introduction

Using Ablowitz, Kaub, Newell, Segur (AKNS) scheme[1] one can obtain coupled Nonlinear Schrödinger (NLS) equations. Extensions of coupled NLS equations have been obtained using a simple Lie algebra[2], a Kac-Moody algebra [3], a Lie superalgebra[3,4], a Virasoro algebra [5] in the literature.

In physics supersymmetry [6] unites bosons and fermions into a single multiplet. Using supersymmetry one can cancel many normally divergent Feynman graphs and one can solve the hierarchy problem in grand unified theories. Supersymmetry also helps to understand the cosmological constant problem in gravity and it reduces the divergences of quantum gravity.

Conformal invariance in two dimensions is a powerful symmetry. Two - dimensional quantum field theories that possess conformal symmetry can be solved exactly by exploiting the conformal symmetry. Conformal symmetry have found remarkable applications in string theory and in the study of critical phenomena in statistical mechanics. N=1 supersymmetric extension of conformal symmetry, called superconformal symmetry promotes string to superstring. The extension of string theory to supersymmetric string theories identified N=1 superconformal algebra (Neveu-Schwarz type[7] and Ramond type [8]) as the symmetry algebras of closed superstrings [9]. The N=1 superconformal algebra with Neveu-Schwarz and Ramond types are two possible super - extensions of the Virasoro algebra [10] for the case of one fermionic current.

In this paper we will obtain super - extensions of coupled NLS equations using N=1 superconformal algebra with Neveu-Schwarz and Ramond types. In sec.2 we will discuss the $osp(1,2)$ superalgebra valued soliton connection and we will obtain coupled super NLS equations. Sec.3 and sec.4 concern the soliton connection for the N=1 superconformal algebra with Neveu-Schwarz type and Ramond type, respectively and we will obtain in these sections two different types of super- extensions of coupled NLS equations.

2 AKNS Scheme with $osp(1,2)$ Superalgebra

In AKNS scheme in 1+1 dimension the connection is defined as

$$\Omega = \begin{pmatrix} i\lambda H_1 + Q^{+1}E_{+1} + Q^{-1}E_{-1} + P^{+\frac{1}{2}}F_{+\frac{1}{2}} + P^{-\frac{1}{2}}F_{-\frac{1}{2}} \\ -AH_1 + B^{+1}E_{+1} + B^{-1}E_{-1} + C^{+\frac{1}{2}}F_{+\frac{1}{2}} + C^{-\frac{1}{2}}F_{-\frac{1}{2}} \end{pmatrix} dx + \begin{pmatrix} \\ \end{pmatrix} dt \quad (1)$$

where H_1, E_{+1}, E_{-1} are bosonic generators and $F_{+\frac{1}{2}}, F_{-\frac{1}{2}}$ are fermionic generators of $osp(1,2)$ superalgebra. These generators have matrix representations

as

$$\begin{aligned} H_1 &= \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}; E_{+1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; E_{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ F_{+\frac{1}{2}} &= \begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}; F_{-\frac{1}{2}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \end{aligned} \quad (2)$$

Also, these generators satisfy the following commutation and anticommutation relations

$$\begin{aligned} [H_1, E_{\pm 1}] &= \pm E_{\pm 1} \\ [E_{+1}, E_{-1}] &= 2H_1 \\ [H_1, F_{\pm \frac{1}{2}}] &= \pm \frac{1}{2} F_{\pm \frac{1}{2}} \\ \{F_{+\frac{1}{2}}, F_{-\frac{1}{2}}\} &= \frac{1}{2} H_1 \\ [E_{\pm 1}, F_{\mp \frac{1}{2}}] &= -F_{\pm \frac{1}{2}} \\ \{F_{\pm \frac{1}{2}}, F_{\pm \frac{1}{2}}\} &= \pm \frac{1}{2} E_{\pm 1} \end{aligned} \quad (3)$$

In Eq.(1) λ is the spectral parameter, $Q^{\pm 1}$ and $P^{\pm \frac{1}{2}}$ are fields depending on space and time, namely x and t , and functions $A, B^{\pm 1}$ and $C^{\pm \frac{1}{2}}$ are x, t and λ dependent.

The integrability condition is given by

$$d\Omega + \Omega \wedge \Omega = 0 \quad (4)$$

By using Eqs.(1) and (4) one can obtain following equations:

$$Q^{+1}_t = B^{+1}_x + i\lambda B^{+1} + Q^{+1}A + \frac{1}{2}P^{+\frac{1}{2}}C^{+\frac{1}{2}} \quad (5)$$

$$Q^{-1}_t = B^{-1}_x - i\lambda B^{-1} + Q^{-1}A - \frac{1}{2}P^{-\frac{1}{2}}C^{-\frac{1}{2}} \quad (6)$$

$$P^{+\frac{1}{2}}_t = C^{+\frac{1}{2}}_x + \frac{i}{2}\lambda C^{+\frac{1}{2}} + \frac{1}{2}P^{+\frac{1}{2}}A + B^{+1}P^{-\frac{1}{2}} - C^{-\frac{1}{2}}Q^{+1} \quad (7)$$

$$P^{-\frac{1}{2}}_t = C^{-\frac{1}{2}}_x - \frac{i}{2}\lambda C^{-\frac{1}{2}} - \frac{1}{2}P^{-\frac{1}{2}}A + B^{-1}P^{+\frac{1}{2}} - C^{+\frac{1}{2}}Q^{-1} \quad (8)$$

$$0 = A_x + 2B^{+1}Q^{-1} - 2B^{-1}Q^{+1} - \frac{1}{2}P^{+\frac{1}{2}}C^{-\frac{1}{2}} - \frac{1}{2}P^{-\frac{1}{2}}C^{+\frac{1}{2}} \quad (9)$$

In AKNS scheme we expand $A, B^{\pm 1}$ and $C^{\pm \frac{1}{2}}$ in terms of positive powers of λ as

$$A = \sum_{n=0}^2 \lambda^n a_n; \quad B^{\pm 1} = \sum_{n=0}^2 \lambda^n b_n^{\pm 1}; \quad C^{\pm \frac{1}{2}} = \sum_{n=0}^2 \lambda^n c_n^{\pm \frac{1}{2}} \quad (10)$$

Inserting Eq.(10) into Eqs.(5-9) gives 15 relations in terms of $a_n, b_n^{\pm 1}$ and $c_n^{\pm \frac{1}{2}}$. By solving these relations we get

$$a_0 = -2iQ^{+1}Q^{-1} - iP^{+\frac{1}{2}}P^{-\frac{1}{2}}; \quad a_1 = 0; \quad a_2 = -2i;$$

$$\begin{aligned}
b_0^{\pm 1} &= \pm i Q^{\pm 1}_x; \quad b_1^{\pm 1} = Q^{\pm 1}; \quad b_2^{\pm 1} = 0 \\
c_0^{\pm \frac{1}{2}} &= \pm 2i P^{\pm \frac{1}{2}}_x; \quad c_1^{\pm \frac{1}{2}} = P^{\pm \frac{1}{2}}; \quad c_2^{\pm \frac{1}{2}} = 0
\end{aligned} \tag{11}$$

By using the relations given by Eq.(11) from Eqs.(5-8) we obtain the coupled super NLS equations as

$$\begin{aligned}
-iQ^{+1}_t &= Q^{+1}_{xx} - 2(Q^{+1})^2 Q^{-1} - P^{+\frac{1}{2}} P^{-\frac{1}{2}} Q^{+1} + P^{+\frac{1}{2}} P_x^{+\frac{1}{2}} \\
iQ^{-1}_t &= Q^{-1}_{xx} - 2(Q^{-1})^2 Q^{+1} - P^{+\frac{1}{2}} P^{-\frac{1}{2}} Q^{-1} - P^{-\frac{1}{2}} P_x^{-\frac{1}{2}} \\
-iP^{+\frac{1}{2}}_t &= 2P^{+\frac{1}{2}}_{xx} + P^{-\frac{1}{2}} Q^{+1}_x + 2Q^{+1} P_x^{-\frac{1}{2}} - Q^{+1} Q^{-1} P^{+\frac{1}{2}} \\
iP^{-\frac{1}{2}}_t &= 2P^{-\frac{1}{2}}_{xx} + P^{+\frac{1}{2}} Q^{-1}_x + 2Q^{-1} P_x^{+\frac{1}{2}} - Q^{+1} Q^{-1} P^{-\frac{1}{2}}
\end{aligned} \tag{12}$$

3 AKNS Scheme with N=1 Superconformal Algebra (Neveu-Schwarz Type)

We generalize the connection given by Eq.(1) as

$$\Omega = \begin{pmatrix} i\lambda L_0 + Q^{+m} L_{+m} + Q^{-m} L_{-m} + P^{+\frac{m}{2}} G_{+\frac{m}{2}} + P^{-\frac{m}{2}} G_{-\frac{m}{2}} \\ -AL_0 + B^{+m} L_{+m} + B^{-m} L_{-m} + C^{+\frac{m}{2}} G_{+\frac{m}{2}} + C^{-\frac{m}{2}} G_{-\frac{m}{2}} \end{pmatrix} dx + dt \tag{13}$$

where L_0 , $L_{\pm m}$ are bosonic generators and $G_{\pm \frac{m}{2}}$ are fermionic generators of centerless N=1 superconformal algebra of Neveu-Schwarz type, namely they satisfy the following commutation and anticommutation relations [6]

$$\begin{aligned}
[L_r, L_s] &= (r-s) L_{r+s} \\
\{G_r, G_s\} &= 2 L_{r+s} \\
[L_r, G_s] &= (\frac{r}{2} - s) G_{r+s}
\end{aligned} \tag{14}$$

Here, $L_{\pm m}$ are generators with positive(negative) integer indices and $G_{\pm \frac{m}{2}}$ are generators with positive(negative) half integer indices. In Eq.(13) we assume summation over the repeated indices. The fields $Q^{\pm m}$ and $P^{\pm \frac{m}{2}}$ are x,t dependent and also functions A, $B^{\pm m}$ and $C^{\pm \frac{m}{2}}$ are x,t and λ dependent.

In N=1 superconformal algebra if we restrict $L_{\pm m}$ to have only L_0 , $L_{\pm 1}$ components and $G_{\pm \frac{m}{2}}$ to have only $G_{\pm \frac{1}{2}}$ components we obtain osp(1,2) algebra given by Eq.(3) with the following definitions:

$$\begin{aligned}
H_1 &= -L_0; \quad E_{+1} = L_{+1}; \quad E_{-1} = -L_{-1} \\
F_{+\frac{1}{2}} &= \frac{1}{2} G_{+\frac{1}{2}}; \quad F_{-\frac{1}{2}} = -\frac{1}{2} G_{-\frac{1}{2}}
\end{aligned} \tag{15}$$

From the integrability condition given by Eq.(4) we obtain

$$Q^{+m}_t = B^{+m}_x - im\lambda B^{+m} - mAQ^m$$

$$\begin{aligned}
& + \sum_{r,s=1}^{\infty} (r-s)B^{+s}Q^{+r}\delta_{r-s,m} + \sum_{\substack{r,s=1 \\ r>s}}^{\infty} (r+s)B^{-s}Q^{+r}\delta_{r-s,m} \\
& - \sum_{\substack{r,s=1 \\ r<s}}^{\infty} (r+s)B^{+s}Q^{-r}\delta_{-r+s,m} + \sum_{\substack{r,s=1 \\ r>s}}^{\infty} 2P^{+\frac{r}{2}}C^{-\frac{s}{2}}\delta_{r-s,2m} \\
& + \sum_{\substack{r,s=1 \\ r<s}}^{\infty} 2P^{-\frac{r}{2}}C^{+\frac{s}{2}}\delta_{-r+s,2m} + \sum_{r,s=1}^{\infty} 2P^{+\frac{r}{2}}C^{+\frac{s}{2}}\delta_{r+s,2m}
\end{aligned} \tag{16}$$

$$\begin{aligned}
& Q^{-m}_t = B^{-m}_x + im\lambda B^{-m} + mAQ^{-m} \\
& - \sum_{r,s=1}^{\infty} (r-s)B^{-s}Q^{-r}\delta_{-r-s,-m} + \sum_{\substack{r,s=1 \\ r<s}}^{\infty} (r+s)B^{-s}Q^{+r}\delta_{r-s,-m} \\
& - \sum_{\substack{r,s=1 \\ r>s}}^{\infty} (r+s)B^{+s}Q^{-r}\delta_{-r+s,-m} + \sum_{\substack{r,s=1 \\ r<s}}^{\infty} 2P^{+\frac{r}{2}}C^{-\frac{s}{2}}\delta_{r-s,-2m} \\
& + \sum_{\substack{r,s=1 \\ r<s}}^{\infty} 2P^{-\frac{r}{2}}C^{+\frac{s}{2}}\delta_{-r+s,-2m} + \sum_{r,s=1}^{\infty} 2P^{-\frac{r}{2}}C^{-\frac{s}{2}}\delta_{-r-s,-2m}
\end{aligned} \tag{17}$$

$$\begin{aligned}
& P^{+\frac{m}{2}}_t = C^{+\frac{m}{2}}_x - \frac{im}{2}\lambda C^{+\frac{m}{2}} - \frac{m}{2}P^{+\frac{m}{2}}A \\
& + \sum_{\substack{r,s=1 \\ 2r>s}}^{\infty} \frac{1}{2}(r+s)Q^{+r}C^{-\frac{s}{2}}\delta_{2r-s,m} - \sum_{\substack{r,s=1 \\ 2r<s}}^{\infty} \frac{1}{2}(r+s)Q^{-r}C^{+\frac{s}{2}}\delta_{-2r+s,m} \\
& + \sum_{r,s=1}^{\infty} \frac{1}{2}(r-s)Q^{+r}C^{+\frac{s}{2}}\delta_{2r+s,m} + \sum_{\substack{r,s=1 \\ r>2s}}^{\infty} \frac{1}{2}(r+s)P^{+\frac{r}{2}}B^{-s}\delta_{r-2s,m} \\
& + \sum_{r,s=1}^{\infty} \frac{1}{2}(r-s)P^{+\frac{r}{2}}B^{+s}\delta_{r+2s,m} - \sum_{\substack{r,s=1 \\ r<2s}}^{\infty} \frac{1}{2}(r+s)P^{-\frac{r}{2}}B^{+s}\delta_{-r+2s,m}
\end{aligned} \tag{18}$$

$$\begin{aligned}
& P^{-\frac{m}{2}}_t = C^{-\frac{m}{2}}_x + \frac{im}{2}\lambda C^{-\frac{m}{2}} + \frac{m}{2}P^{-\frac{m}{2}}A \\
& + \sum_{\substack{r,s=1 \\ 2r<s}}^{\infty} \frac{1}{2}(r+s)Q^{+r}C^{-\frac{s}{2}}\delta_{2r-s,-m} - \sum_{\substack{r,s=1 \\ 2r>s}}^{\infty} \frac{1}{2}(r+s)Q^{-r}C^{+\frac{s}{2}}\delta_{-2r+s,-m} \\
& - \sum_{r,s=1}^{\infty} \frac{1}{2}(r-s)Q^{-r}C^{-\frac{s}{2}}\delta_{-2r-s,-m} + \sum_{\substack{r,s=1 \\ r<2s}}^{\infty} \frac{1}{2}(r+s)P^{+\frac{r}{2}}B^{-s}\delta_{r-2s,-m}
\end{aligned} \tag{19}$$

$$- \sum_{r,s=1}^{\infty} \frac{1}{2} (r-s) P^{-\frac{r}{2}} B^{-s} \delta_{-r-2s,-m} - \sum_{\substack{r,s=1 \\ r>2s}}^{\infty} \frac{1}{2} (r+s) P^{-\frac{r}{2}} B^{+s} \delta_{-r+2s,-m}$$

and

$$0 = -A_x + 2 \sum_{r=1}^{\infty} (r B^{-r} Q^{+r} - r B^{+r} Q^{-r} + P^{+\frac{r}{2}} C^{-\frac{r}{2}} + P^{-\frac{r}{2}} C^{+\frac{r}{2}}) \quad (20)$$

In AKNS scheme we expand $A, B^{\pm m}$ and $C^{\pm \frac{m}{2}}$ in terms of the positive powers of λ as

$$A = \sum_{n=0}^2 \lambda^n a_n; \quad B^{\pm m} = \sum_{n=0}^2 \lambda^n b_n^{\pm m}; \quad C^{\pm \frac{m}{2}} = \sum_{n=0}^2 \lambda^n c_n^{\pm \frac{m}{2}} \quad (21)$$

Inserting Eq.(21) into Eqs.(16-20) gives 15 relations in terms of $a_n, b_n^{\pm m}$ and $c_n^{\pm \frac{m}{2}}$ ($n=0,1,2$). By solving these relations we get

$$\begin{aligned} a_0 &= 2i \sum_{r=1}^{\infty} Q^{+r} Q^{-r} + \frac{4i}{m} \sum_{r=1}^{\infty} P^{+\frac{r}{2}} P^{-\frac{r}{2}}; \quad a_1 = \text{const.} = a_{10}; \quad a_2 = -i; \\ b_0^{\pm m} &= \mp \frac{i}{m} Q^{\pm m}_x + a_{10} i Q^{\pm m}; \quad b_1^{\pm m} = Q^{\pm m}; \quad b_2^{\pm m} = 0 \\ c_0^{\pm \frac{m}{2}} &= a_{10} i P^{\pm \frac{m}{2}} \mp \frac{2i}{m} P^{\pm \frac{m}{2}}_x; \quad c_1^{\pm \frac{m}{2}} = P^{\pm \frac{m}{2}}; \quad c_2^{\pm \frac{m}{2}} = 0 \end{aligned} \quad (22)$$

By using the relations given by Eq.(22) from Eqs.(16-19) we obtain the coupled super NLS equations as

$$\begin{aligned} Q^{+m}_t &= \frac{-i}{m} Q^{+m}_{xx} + i a_{10} Q^{+m}_x \\ &- 2im Q^{+m} \left(\sum_{r=1}^{\infty} Q^{+r} Q^{-r} \right) - 4im Q^{+m} \left(\sum_{r=1}^{\infty} \left[\frac{1}{r} \right] P^{+\frac{r}{2}} P^{-\frac{r}{2}} \right) \\ &- i \sum_{\substack{r=1 \\ r < m}}^{\infty} \left[\frac{2r-m}{m-r} \right] Q^{+r} Q_x^{+(m-r)} - i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left[\frac{2r-m}{m-r} \right] Q^{+r} Q_x^{-(r-m)} \\ &+ i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left[\frac{2r-m}{r} \right] Q^{-(r-m)} Q_x^{+r} + 4i \sum_{\substack{r=1 \\ r > 2m}}^{\infty} \left[\frac{1}{r-2m} \right] P^{+\frac{r}{2}} P_x^{-(\frac{r}{2}-m)} \\ &- 4i \sum_{\substack{r=1 \\ r > 2m}}^{\infty} \left[\frac{1}{r} \right] P^{-(\frac{r}{2}-m)} P_x^{+\frac{r}{2}} - 4i \sum_{\substack{r=1 \\ r < 2m}}^{\infty} \left[\frac{1}{2m-r} \right] P^{+\frac{r}{2}} P_x^{+(m-\frac{r}{2})} \end{aligned} \quad (23)$$

$$\begin{aligned}
Q^{-m}_t &= \frac{i}{m} Q^{-m}_{xx} + ia_{10} Q^{-m}_x \\
&+ 2im Q^{-m} \left(\sum_{r=1}^{\infty} Q^{+r} Q^{-r} \right) + 4im Q^{-m} \left(\sum_{r=1}^{\infty} \left[\frac{1}{r} \right] P^{+\frac{r}{2}} P^{-\frac{r}{2}} \right) \\
&+ i \sum_{\substack{r=1 \\ r < m}}^{\infty} \left[\frac{2r-m}{r-m} \right] Q^{-r} Q_x^{+(r-m)} + i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left[\frac{2r-m}{r-m} \right] Q^{-r} Q_x^{+(r-m)} \\
&+ i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left[\frac{2r-m}{r} \right] Q^{+(r-m)} Q_x^{-r} + 4i \sum_{\substack{r=1 \\ r > 2m}}^{\infty} \left[\frac{1}{r} \right] P^{+(\frac{r}{2}-m)} P_x^{-\frac{r}{2}} \quad (24) \\
&- 4i \sum_{\substack{r=1 \\ r > 2m}}^{\infty} \left[\frac{1}{r-2m} \right] P^{-\frac{r}{2}} P_x^{+(\frac{r}{2}-m)} - 4i \sum_{\substack{r=1 \\ r < 2m}}^{\infty} \left[\frac{1}{r-2m} \right] P^{-\frac{r}{2}} P_x^{+(\frac{r}{2}-m)}
\end{aligned}$$

$$\begin{aligned}
P^{+\frac{m}{2}}_t &= \frac{-2i}{m} P^{+\frac{m}{2}}_{xx} + ia_{10} P^{+\frac{m}{2}}_x \\
&- im P^{+\frac{m}{2}} \left(\sum_{r=1}^{\infty} Q^{+r} Q^{-r} \right) - 2im P^{+\frac{m}{2}} \left(\sum_{r=1}^{\infty} \left[\frac{1}{r} \right] P^{+\frac{r}{2}} P^{-\frac{r}{2}} \right) \\
&+ \frac{i}{2} \sum_{\substack{r=1 \\ r > m}}^{\infty} \left[\frac{3r-m}{r-m} \right] P^{+\frac{r}{2}} Q_x^{-(\frac{r}{2}-\frac{m}{2})} + \frac{i}{2} \sum_{\substack{r=1 \\ r > \frac{m}{2}}}^{\infty} \left[\frac{3r-m}{r} \right] P^{-(r-\frac{m}{2})} Q_x^{+r} \\
&+ \frac{i}{2} \sum_{\substack{r=1 \\ r < m}}^{\infty} \left[\frac{3r-m}{r-m} \right] P^{+\frac{r}{2}} Q_x^{-(\frac{m}{2}-\frac{r}{2})} + i \sum_{\substack{r=1 \\ r > \frac{m}{2}}}^{\infty} \left[\frac{3r-m}{2r-m} \right] Q^{+\frac{r}{2}} P_x^{-(r-\frac{m}{2})} \quad (25) \\
&+ i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left[\frac{3r-m}{2r} \right] Q^{-(\frac{r}{2}-\frac{m}{2})} P_x^{+\frac{r}{2}} + i \sum_{\substack{r=1 \\ r < \frac{m}{2}}}^{\infty} \left[\frac{3r-m}{2r-m} \right] Q^{+r} P_x^{-(\frac{m}{2}-r)}
\end{aligned}$$

and

$$\begin{aligned}
P^{-\frac{m}{2}}_t &= \frac{2i}{m} P^{-\frac{m}{2}}_{xx} + ia_{10} P^{-\frac{m}{2}}_x \\
&+ im P^{-\frac{m}{2}} \left(\sum_{r=1}^{\infty} Q^{+r} Q^{-r} \right) + 2im P^{-\frac{m}{2}} \left(\sum_{r=1}^{\infty} \left[\frac{1}{r} \right] P^{+\frac{r}{2}} P^{-\frac{r}{2}} \right) \\
&+ \frac{i}{2} \sum_{\substack{r=1 \\ r > \frac{m}{2}}}^{\infty} \left[\frac{3r-m}{r} \right] P^{+(r-\frac{m}{2})} Q_x^{-r} + \frac{i}{2} \sum_{\substack{r=1 \\ r > m}}^{\infty} \left[\frac{3r-m}{r-m} \right] P^{-\frac{r}{2}} Q_x^{+(\frac{r}{2}-\frac{m}{2})} \\
&+ \frac{i}{2} \sum_{\substack{r=1 \\ r < m}}^{\infty} \left[\frac{3r-m}{r-m} \right] P^{-\frac{r}{2}} Q_x^{+(\frac{r}{2}-\frac{m}{2})} + i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left[\frac{3r-m}{2r} \right] Q^{+(\frac{r}{2}-\frac{m}{2})} P_x^{-\frac{r}{2}} \quad (26)
\end{aligned}$$

$$+i \sum_{\substack{r=1 \\ r > \frac{m}{2}}}^{\infty} \left[\frac{3r-m}{2r-m} \right] Q^{-r} P_x^{+(r-\frac{m}{2})} + i \sum_{\substack{r=1 \\ r < \frac{m}{2}}}^{\infty} \left[\frac{3r-m}{2r-m} \right] Q^{-r} P_x^{+(r-\frac{m}{2})}$$

4 AKNS Scheme with N=1 Superconformal Algebra (Ramond Type)

We take the soliton connection as

$$\Omega = \begin{pmatrix} i\lambda L_0 + Q^{+m} L_{+m} + Q^{-m} L_{-m} + P^{+m} G_{+m} + P^{-m} G_{-m} \\ AL_0 + DG_0 + B^{+m} L_{+m} + B^{-m} L_{-m} + C^{+m} G_{+m} + C^{-m} G_{-m} \end{pmatrix} dx + dt \quad (27)$$

where L_0 , $L_{\pm m}$ are bosonic generators and $G_{\pm m}$ are fermionic generators of centerless N=1 superconformal algebra of Ramond type, namely they satisfy the following commutation and anticommutation relations [6]

$$\begin{aligned} [L_r, L_s] &= (r-s) L_{r+s} \\ \{G_r, G_s\} &= 2 L_{r+s} \\ [L_r, G_s] &= (\frac{r}{2} - s) G_{r+s} \end{aligned} \quad (28)$$

Here, $L_{\pm m}$ and $G_{\pm m}$ are generators with positive(negative) integer indices. In Eq.(27) we assume summation over the repeated indices. The fields $Q^{\pm m}$ and $P^{\pm m}$ are x,t dependent and also functions A,D, $B^{\pm m}$ and $C^{\pm m}$ are x,t and λ dependent.

From the integrability condition given by Eq.(4) we obtain

$$\begin{aligned} Q^{+m}_t &= B^{+m}_x - im\lambda B^{+m} - mAQ^m - 2P^{+m}D \\ &+ \sum_{r,s=1}^{\infty} (2P^{+s}C^{+r} + (s-r)Q^{+s}B^{+r}) \delta_{r+s,m} \\ &+ \sum_{\substack{r,s=1 \\ r < s}}^{\infty} (2P^{-r}C^{+s} - (r+s)Q^{-r}B^{+s}) \delta_{-r+s,m} \\ &+ \sum_{\substack{r,s=1 \\ r > s}}^{\infty} (2P^{+r}C^{-s} + (r+s)Q^{+r}B^{-s}) \delta_{r-s,m} \\ Q^{-m}_t &= B^{-m}_x + im\lambda B^{-m} + mAQ^{-m} - 2P^{-m}D \\ &+ \sum_{r,s=1}^{\infty} (2P^{+s}C^{+r} + (s-r)Q^{+s}B^{+r}) \delta_{r+s,m} \end{aligned} \quad (29)$$

$$+ \sum_{\substack{r,s=1 \\ r < s}}^{\infty} (2P^{-r}C^{+s} - (r+s)Q^{-r}B^{+s}) \delta_{-r+s,m} \quad (30)$$

$$+ \sum_{\substack{r,s=1 \\ r > s}}^{\infty} (2P^{+r}C^{-s} + (r+s)Q^{+r}B^{-s}) \delta_{r-s,m}$$

$$P^{+m}_t = C^{+m}_x - im\lambda C^{+m} - mP^{+m}A - \frac{m}{2}Q^{+m}D$$

$$- \frac{1}{2} \sum_{r,s=1}^{\infty} ((r-2s)P^{+r}B^{+s} + (2r-s)Q^{+s}C^{+r}) \delta_{r+s,m}$$

$$- \frac{1}{2} \sum_{\substack{r,s=1 \\ r < s}}^{\infty} ((2r+s)P^{-r}B^{+s} + (r+2s)Q^{-r}C^{+s}) \delta_{-r+s,m} \quad (31)$$

$$+ \frac{1}{2} \sum_{\substack{r,s=1 \\ r > s}}^{\infty} ((2r+s)P^{+r}B^{-s} + (r+2s)Q^{+r}C^{-s}) \delta_{r-s,m}$$

$$P^{-m}_t = C^{-m}_x + im\lambda C^{-m} + mP^{-m}A + \frac{m}{2}Q^{-m}D$$

$$\frac{1}{2} \sum_{r,s=1}^{\infty} ((s-2r)P^{-r}B^{-s} + (s-2r)Q^{-r}C^{-s}) \delta_{-r-s,-m}$$

$$- \frac{1}{2} \sum_{\substack{r,s=1 \\ r > s}}^{\infty} ((r+2s)P^{-r}B^{+s} + (r+2s)Q^{-r}C^{+s}) \delta_{-r+s,-m} \quad (32)$$

$$+ \frac{1}{2} \sum_{\substack{r,s=1 \\ r < s}}^{\infty} ((2r+s)P^{+r}B^{-s} + (r+2s)Q^{+r}C^{-s}) \delta_{r-s,-m}$$

$$0 = -A_x + 2 \sum_{r=1}^{\infty} (rB^{-r}Q^{+r} - rB^{+r}Q^{-r} + P^{+r}C^{-r} + P^{-r}C^{+r}) \quad (33)$$

and

$$0 = -D_x + \frac{3}{2} \sum_{r=1}^{\infty} r (P^{+r}B^{-r} - P^{-r}B^{+r} - Q^{-r}C^{+r} + Q^{+r}C^{-r}) \quad (34)$$

In AKNS scheme we expand $A, D, B^{\pm m}$ and $C^{\pm m}$ in terms of the positive powers of λ as

$$A = \sum_{n=0}^2 \lambda^n a_n; \quad D = \sum_{n=0}^2 \lambda^n d_n; \quad B^{\pm m} = \sum_{n=0}^2 \lambda^n b_n^{\pm m}; \quad C^{\pm m} = \sum_{n=0}^2 \lambda^n c_n^{\pm m} \quad (35)$$

Inserting Eq.(35) into Eqs.(29-34) gives 18 relations in terms of $a_n, d_n, b_n^{\pm m}$ and $c_n^{\pm m}$ ($n=0,1,2$). By solving these relations we get

$$\begin{aligned} a_0 &= 2i \sum_{r=1}^{\infty} Q^{+r} Q^{-r} + \frac{2i}{m} \sum_{r=1}^{\infty} P^{+r} P^{-r}; \quad a_1 = 0; \quad a_2 = -i; \\ b_0^{\pm m} &= \mp \frac{i}{m} Q^{\pm m}_x; \quad b_1^{\pm m} = Q^{\pm m}; \quad b_2^{\pm m} = 0 \\ c_0^{\pm m} &= \mp \frac{i}{m} P^{\pm m}_x; \quad c_1^{\pm m} = P^{\pm m}; \quad c_2^{\pm m} = 0 \\ d_0 &= \frac{3i}{2} \sum_{r=1}^{\infty} P^{-r} Q^{+r} + \frac{3i}{2} \sum_{r=1}^{\infty} Q^{+r} P^{-r}; \quad d_1 = 0; \quad d_2 = 0; \end{aligned} \quad (36)$$

By using the relations given by Eq.(36) from Eqs.(29-32) we obtain the coupled super NLS equations as

$$\begin{aligned} Q^{+m}_t &= \frac{-i}{m} Q^{+m}_{xx} - 2im Q^{+m} \left(\sum_{r=1}^{\infty} Q^{+r} Q^{-r} \right) \\ &- 4im Q^{+m} \left(\sum_{r=1}^{\infty} \left[\frac{1}{r} \right] P^{+\frac{r}{2}} P^{-\frac{r}{2}} \right) - 3i \sum_{r=1}^{\infty} (Q^{+r} P^{-r} + Q^{-r} P^{+r}) \\ &- i \sum_{\substack{r=1 \\ r < m}}^{\infty} \left(\left[\frac{2}{r} \right] P^{+(m-r)} P^{+r}_x - \left[\frac{2r-m}{r} \right] Q^{+(m-r)} Q^{+r}_x \right) \\ &+ i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left(\left[\frac{2}{r-m} \right] P^{+r} P^{-(r-m)}_x + \left[\frac{2r-m}{r-m} \right] Q^{+r} Q^{-(r-m)}_x \right) \\ &- i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left(\left[\frac{2}{r} \right] P^{-(r-m)} P^{+r}_x - \left[\frac{2r-m}{r} \right] Q^{-(r-m)} Q^{+r}_x \right) \\ Q^{-m}_t &= \frac{i}{m} Q^{-m}_{xx} + 2im Q^{-m} \left(\sum_{r=1}^{\infty} Q^{+r} Q^{-r} \right) \\ &+ 4im Q^{-m} \left(\sum_{r=1}^{\infty} \left[\frac{1}{r} \right] P^{+\frac{r}{2}} P^{-\frac{r}{2}} \right) + 3i \sum_{r=1}^{\infty} (Q^{+r} P^{-r} + Q^{-r} P^{+r}) \end{aligned} \quad (37)$$

$$\begin{aligned}
& -i \sum_{\substack{r=1 \\ r < m}}^{\infty} \left(\left[\frac{2}{r-m} \right] P^{-r} P_x^{+(r-m)} + \left[\frac{2r-m}{r-m} \right] Q^{-r} Q_x^{+(r-m)} \right) \\
& + i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left(\left[\frac{2}{r} \right] P^{+(r-m)} P_x^{-r} + \left[\frac{2r-m}{r} \right] Q^{+(r-m)} Q_x^{-r} \right)
\end{aligned} \tag{38}$$

$$\begin{aligned}
& -i \sum_{\substack{r=1 \\ r > m}}^{\infty} \left(\left[\frac{2}{r-m} \right] P^{-r} P_x^{+(r-m)} - \left[\frac{2r-m}{r-m} \right] Q^{-r} Q_x^{+(r-m)} \right) \\
& P^{+m}_t = \frac{-i}{m} P^{+m}_{xx} - 2im P^{+m} \left(\sum_{r=1}^{\infty} Q^{+r} Q^{-r} \right) \\
& - 2im P^{+m} \left(\sum_{r=1}^{\infty} \left[\frac{1}{r} \right] P^{+\frac{r}{2}} P^{-\frac{r}{2}} \right) - \frac{3}{4} im Q^{+m} \sum_{r=1}^{\infty} (Q^{+r} P^{-r} + Q^{-r} P^{+r}) \\
& + \frac{i}{2} \sum_{\substack{r=1 \\ r < m}}^{\infty} \left(\left[\frac{2r-m}{r} \right] P^{+(m-r)} Q_x^{+r} + \left[\frac{r-m}{r} \right] Q^{+(m-r)} P_x^{+r} \right)
\end{aligned} \tag{39}$$

$$\begin{aligned}
& + \frac{i}{2} \sum_{\substack{r=1 \\ r > m}}^{\infty} \left(\left[\frac{3r-m}{r-m} \right] P^{+r} Q_x^{-(r-m)} + \left[\frac{3r-2m}{r-m} \right] Q^{+r} P_x^{-(r-m)} \right) \\
& + \frac{i}{2} \sum_{\substack{r=1 \\ r > m}}^{\infty} \left(\left[\frac{3r-m}{r} \right] P^{-(r-m)} Q_x^{+r} + \left[\frac{3r-m}{r} \right] Q^{-(r-m)} P_x^{+r} \right) \\
& P^{-m}_t = \frac{i}{m} P^{-m}_{xx} + 2im P^{-m} \left(\sum_{r=1}^{\infty} Q^{+r} Q^{-r} \right) \\
& + 2im P^{-m} \left(\sum_{r=1}^{\infty} \left[\frac{1}{r} \right] P^{+\frac{r}{2}} P^{-\frac{r}{2}} \right) + \frac{3}{4} im Q^{-m} \sum_{r=1}^{\infty} (Q^{+r} P^{-r} + Q^{-r} P^{+r}) \\
& + \frac{i}{2} \sum_{\substack{r=1 \\ r < m}}^{\infty} \left(\left[\frac{3r-m}{r-m} \right] P^{-r} Q_x^{+(r-m)} + \left[\frac{3r-2m}{r-m} \right] Q^{-r} P_x^{+(r-m)} \right) \\
& + \frac{i}{2} \sum_{\substack{r=1 \\ r > m}}^{\infty} \left(\left[\frac{3r-m}{r} \right] P^{+(r-m)} Q_x^{-r} + \left[\frac{3r-m}{r} \right] Q^{+(r-m)} P_x^{-r} \right) \\
& + \frac{i}{2} \sum_{\substack{r=1 \\ r > m}}^{\infty} \left(\left[\frac{3r-m}{r-m} \right] P^{-r} Q_x^{+(r-m)} + \left[\frac{3r-m}{r-m} \right] Q^{-r} P_x^{+(r-m)} \right)
\end{aligned} \tag{40}$$

5 Conclusions

Using AKNS scheme and N=1 superconformal algebra of Neveu-Schwarz and Ramond types we obtain two different new super- extensions of coupled Nonlinear Schrödinger equations.

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